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Content knowledge and pedagogical content knowledge in the tenth grade mathematics textbook of West Bengal board of secondary education

Dr. Subir Sen

Assistant Professor, B. Ed. Department, Kalna College, The University of Burdwan, West Bengal, India

Abstract
Present work deals with an analysis of content knowledge (CK) and pedagogical content knowledge (PCK) in tenth grade mathematics textbook of West Bengal Board of Secondary Education (WBBSE), West Bengal, India. Concepts which are explored in the algebra together with necessary concepts which should be included are also discussed. An analysis on textual presentation and exercises supplied in the textbook of WBBSE is done. For improvement of the textbook necessary recommendations are argued.

Keywords: Mathematics textbook, Algebra, Content knowledge, Pedagogical Content Knowledge

Introduction

At present more than ten Lac (10, 00,000) students of West Bengal, India have been reading in tenth grade in every year under WBBSE. To motivate them in the context of mathematics learning is a crucial factor. Learning materials form an indispensable ingredient in the process of learning acquisition. Therefore, serious attention is to be laid on the scientific preparation of the text book materials so that motivation level of the learners may be crystallized at the outset of learning. If a cautious survey of the existing Mathematics textbooks designed by the WBBSE is made, it may lay bare not only some positive facets of the books but also some shortcomings and anomalies that need to be seriously taken into notice by the respective board so thatMathematics learning may be made more feasible at the secondary level. Therefore, any positive change in the textbooks will not only enrich the cognitive domain but also affective and psychomotor domains of the learners belonging to the various levels of learning concerned.

Shulman (1987) defines seven categories to provide a framework for teacher knowledge which are:

1. Content knowledge
2. General pedagogical knowledge eg classroom control, using group work
3. Pedagogical content knowledge
4. Curriculum knowledge
5. Knowledge of learners and their characteristics
6. Knowledge of educational contexts eg schools and the wider community
7. Knowledge of educational ends purposes and values

Shulman (1987) identified seven domains of teacher knowledge, one of which is pedagogical content knowledge. He explained why he identified pedagogical content knowledge as a knowledge domain for teachers as follows:

Pedagogical content knowledge is of special interest because it identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented and adapted to the diverse interests and abilities of learners, and presented for instruction.

Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue. (p. 8) Shulman claimed that pedagogical content knowledge is a distinct body of knowledge even though knowledge of content and knowledge of pedagogy contribute to it. He also noted that pedagogical content knowledge includes knowledge of learners, knowledge of educational context, and knowledge of instructional materials.

Tamir (1988) [28] made a distinction between general pedagogical knowledge and subject-matter-specific pedagogical knowledge. He claimed that each type of knowledge is composed of four categories-namely, student, curriculum, instruction, and evaluation but they have different meanings in each domain. He also identified teachers' skills in diagnosing students' conceptual difficulties in a given topic and their knowledge about effective use of instructional tools as subject-matter-specific pedagogical knowledge.

Ball and Bass (2000) identified teachers' knowledge of students' difficulties and appropriate teaching strategies to eliminate those difficulties as part of teachers' pedagogical content knowledge. They defined pedagogical content knowledge as follows:

'Pedagogical content knowledge is a special form of knowledge that bundles mathematical knowledge with knowledge of learners, learning, and pedagogy. These bundles offer a crucial resource for teaching mathematics, for they can help the teacher anticipate what students might have trouble learning, and have ready alternative models or explanations to mediate those difficulties (p. 88).'

Wang Wei Sönnerhed (2011) [30] studied algebra textbook for CK and PCK and wrote 'The primary aim of the study is to explore what pedagogical content knowledge regarding solving quadratic equations that is embedded in mathematics textbooks. The secondary aim is to analyze the algebra content related to solving quadratic equations from the perspective of mathematics as a discipline in relation to

algebra history. It is about what one can find in the textbook rather than how the textbook is used in the classroom (p-5).'

According to Huang & Kulm (2012) [12] 'study examined prospective middle grade mathematics teachers' knowledge of algebra for teaching with a focus on knowledge for teaching the concept of function. 115 prospective teachers from an interdisciplinary program for mathematics and science middle teacher preparation at a large public university in the USA participated in a survey. It was found that the participants had relatively limited knowledge of algebra for teaching. They also revealed weakness in selecting appropriate perspectives of the concept of function and flexibly using representations of quadratic functions. They made numerous mistakes in solving quadratic or irrational equations and in algebraic manipulation and reasoning.

A systematic review of the way PCK was conceptualized and (empirically) studied in mathematics education research was done by Depaepe *et al*. (2013) [7]. According to them, 'Based on a systematic search in the databases Eric, PsycInfo and Web of Science 60 articles were reviewed. We identified different conceptualizations of PCK that in turn had a differential influence on the methods used in the study of PCK.'

Booth *et al*. (2013) [3] examined whether correct and incorrect examples with prompts for self-explanation can be effective for improving students' conceptual understanding and procedural skill in Algebra when combined with guided practice. McCrory & Stylianides (2014) [16] explored how reasoning-and-proving is treated in the 16 extant textbooks written for mathematics courses for future elementary teachers in the United States to offer insight into the opportunities designed for them to develop knowledge about reasoning-and-proving. Bieda *et al*. (2014) [2] analyzed seven upper elementary (ages 9–11) mathematics textbooks published in the U.S., focusing specifically on reasoning-and-proving opportunities in written tasks, and found that the average percentage of such tasks was 3.7%.

Shepherd & Sande (2014) [26] designed to begin to understand how mathematically more advanced readers read for understanding in mathematical exposition as it appears in textbooks compared to first-year undergraduate students. Otten *et al*. (2014) [17] analyzed the reasoning-and-proving opportunities in six U.S. geometry textbooks, giving particular attention to the chapter that introduced proof. Kollosche (2014) [14] discussed how logic and calculation are linked to epistemology, spirituality and politics; how mathematics education can be understood as an institution for a mathematical enculturation; and how, therefore, mathematics education necessarily (re)produces techniques of power which privilege some children while disadvantaging others. Planas & Setati-Phakeng (2014) [20] expand their prior work on mathematics education in contexts of language diversity by elaborating on the three perspectives on language described by Ruiz (NABE J 8(2):15–34, 1984): language-as-right, language-as-resource, and language-as-problem.

Paolucci (2015) [18] reported that after completing a degree which includes advanced mathematical studies, many prospective teachers' beliefs still reflect limited interpretations of key terminology and do not value the theoretical and conceptual network underpinning the rules and procedures of secondary mathematics. Many of their beliefs about the nature of mathematics also fail to recognize its capacity to stimulate analytical thought and creativity. A study of upper secondary students' changes in beliefs/views about (or images of) mathematics as a (scientific) discipline is discussed by Jankvist (2015) [13]. As a final result of these detected changes, a small model for (or definition of) students' reflected images of mathematics as a discipline is proposed by the author.

Lijanporn & Khlaisang (2015) [15] developed the activity-based learning model using educational mobile application to enhance discipline of elementary school students. The model was developed based on the review of literature and the experts' interview. Zahner (2015) [31] analyzed of mathematical reasoning rooted in Cultural Historical Activity Theory to examine how mathematical discourse and student reasoning about linear functions developed across 3 weeks in a ninth grade bilingual algebra class.

Sen and Samanta (2015a, 2015b, 2015c, 2015d) [22, 23, 24, 25] analyzed content knowledge and pedagogical content knowledge in sixth, seventh, eighth and ninth grade mathematics textbook of West Bengal Board of Secondary Education, West Bengal, India. According to them 'It seems that the algebra content of the text book demands slide modifications. Lastly it is suggested that there should be a concept summary listed at the end of the text and an exercise containing problems on every concept at the extreme end of the units discussed before for correlation and evaluation of concepts.'

Objectives of the study

The objectives of this study is primarily to explore what pedagogical content knowledge regarding algebra, is embedded in the mathematics textbooks used for WBBSE. The study relates to both algebra content and pedagogical content knowledge in the textbooks. An important step and a secondary objective of my study is to analyze the algebra content presented in the textbooks. It is about what one can find in the textbook rather than how the textbook is used in the classroom. This study reflects an analytic interest of algebra content knowledge as subject matter content knowledge. In order to combine these two objectives, I use the CK-PCK framework to analyze the algebra content in the textbook. I have framed the following objectives to explore my area of investigation.

- To explore and analyze what content knowledge regarding algebra, is embedded in the mathematics textbooks
- To relate algebra and pedagogical content knowledge in the textbooks
- To modify or extend concepts which are explored in the textbooks
- To suggest new concepts which may be incorporated in the textbooks

Methodology

To explore the nature of Content Knowledge (CK) represented in the textbook and expected Pedagogical Content Knowledge (PCK) in the framework, it is necessary to discuss Van Dormolin's (1986) classification of teaching perspectives and learning perspectives of Schmidt *et al* (1997) [21].

Based on their classification and the CK-PCK overall framework one may consider the following criteria for

analyzing algebra content textual presentation as follows:

1. Consistency and clearness of Mathematical content: A mathematical text should be consistent and clear to the reader. "There must be no errors, either of computation or of logic. Proofs might be incomplete, but not false. Conventions must be used consistently. […] the content must be clear to the intended reader." (Van Dormolen, 1986, p. 151).
2. Mathematical theoretical aspects: This criterion concerns knowledge elements such as mathematical theorems, rules, definitions, methods and conventions. Such mathematical knowledge is called "kernels" (Van Dormolen, 1986, p. 146)
3. Mathematical content development and connections: This criterion is based on the classification of Schmidt *et al*. (1997) [21]. By means of this criterion, one may investigate how mathematical content topics relate to each other in the chapter of algebra. The aim is to explore the embedded teaching trajectory related to text.
4. Mathematical representations and applications: This category often reflects different views. A formalistic view regards mathematics as a set of concepts, rules, theorems and structures. Mathematics applications are often regarded as informal view. In an informal view students are encouraged to engage in activities like generalizing, classifying, formalizing, ordering, abstracting, exploring patterns and so on, and new ideas are encouraged (De Lange, 1996; Freudenthal, 1991; Goldin, 2008; Pepin *et al*., 2001; Van Dormolen, 1986; Vergnaud, 1987) [6, 8, 9, 19, 29].
5. Language use: In which way are mathematical theorems, definitions, and rules explained and illustrated: formally in a mathematical language or pedagogically in combination with everyday language, in order to make sense for a student reader.
6. To analyze different kinds of mathematics exercises, activities and problems as well as tests in the textbook, it is important to analyzing mathematics tasks in the textbooks (Brändström, 2005) [4]. One may consider the following points:
   A. Routine exercises refer to the kind of exercises that require students to use newly presented mathematical concepts, rules or algorithmic procedures illustrated in examples, in order to get familiar with the content. This kind of exercises is often at a basic level and requires simple and similar operations or reasoning to those just presented.
   B. Exercises that require students to evaluate, analyze and reason mathematically instead of merely computing mechanically (Brändström, 2005) [4]. Such exercises intend to encourage students to understand the integration of mathematics concepts and procedures (Hiebert & Carpenter, 2007; Hiebert & Lefevre, 1986) [10, 11].
   C. Exercises that are related to real world contexts. Such exercises are often word problems (or called real world problems) and the pedagogical reason of using them is to bring reality into the mathematics classroom, to create occasions for learning and practicing the different aspects of applied problem solving without the practical contact with the real world situation (Chapman, 2006). They reflect the view of mathematics applications in real-life situations (De Lange, 1996; Freudenthal, 1991; Goldin, 2008; Pepin *et al*., 2001; Van Dormolen, 1986; Vergnaud, 1987) [6, 8, 9, 19, 29].

Analyzing Procedure

To analyze the algebraic contents of the textbooks of WBBSE following procedure is followed. Five round analyses is done by repeating the following steps.

i) Identification of concepts presented in the textbook.
ii) Development of the text.
iii) Correlation among the concepts presented in the text.
iv) Presentation of the text: consistency, clearness and use of language.
v) Analysis of the theoretical aspects.
vi) Application of the rules, theorems, concepts in mathematical as well as word problems in examples.
vii) Analysis of exercises.

Textbook of 10th grade is taken for analysis. I have identified the units in which algebra contents are presented. In this textbook there are two such units viz. unit I (Quadratic Equation with one variable) and unit V (Ratio and Proportion). After that, unit I is considered for analysis and five round analysis is done by considering the steps mentioned before. Same procedure is followed for unit V.

Results and Discussions

Name of the tenth grade mathematics text book of WBBSE is 'Ganit Prakash (class X)'.

In this book, content of unit I (p. 1-30) is Quadratic Equation with one variable.

Representation of the text in this unit is as follows:

- Construction of quadratic equation (coefficients are real numbers) under certain condition is done with the help of real life situation.
- Statement that $ax^2+bx+c=0$ *where* a,b,c *reals and* $a\neq 0$ represents quadratic equation in one variable.
- More examples on construction of quadratic equation and its verification by the criteria $ax^2+bx+c=0$ *where* a,b,c *reals and* $a\neq 0$ are stated.
- Transformation of higher order equation into quadratic equation is shown by an exercise as follows: Find out for which power of the variable $x^6-x^3-2=0$ can be transform into a quadratic equation.
- Solution of quadratic equation by factorization method is illustrated. The terms 'root' and 'solution' are stated.
- It is mentioned that zero of a polynomial ax^2+bx+c *where* a,b,c *reals and* $a\neq 0$ is actually the roots of the quadratic equation $ax^2+bx+c=0$ *where* a,b,c *reals and* $a\neq 0$.
- The concept 'as a quadratic equation has two zeros, therefore it has two roots' is mentioned with the help of fill in the blanks.
- Applications of quadratic equation in word problems are illustrated.
- To determine the nature of the roots of a quadratic equation $ax^2+bx+c=0$ *where* a,b,c *reals* $a\neq 0$ by

representing it by $(x+p)^2-q^2=0$ $(p,q \text{ reals})$. This is actually method of completing square.

- An activity is taken by representing $x^2+6x+5=0$ *by* $(x+3)^2-2^2=0$
- Application of completing square method is illustrated.
- An illustration is made with the help of real examples that some quadratic equation has no real roots.
- Method of Shridhar Acherya is applied to compute roots of a quadratic equation. It is mentioned that the only real roots will be calculated by this method.

[$ax^2+bx+c=0,\ a,b,c$ *reals* $a\neq 0$,

$\Rightarrow x=\dfrac{-b+\sqrt{b^2-4ac}}{2a}$ *and* $\dfrac{-b-\sqrt{b^2-4ac}}{2a}$ and solution will be done only when $b^2-4ac\geq 0$].

- A historical reference of method of Shridhar Acharya is discussed.
- If two values of x satisfy a quadratic equation in one variable then it is certain that they are the roots of the given equation. In this case it is also illustrated that the word problem, if the solution does not satisfy the condition of the problem then it will not be accepted as solution.
- Examples are illustrated with the help of Shridhar Acharyas' method of solution of quadratic equation.
- Nature of the roots of a quadratic equation $ax^2+bx+c=0$ *where* a,b,c *reals and* $a\neq 0$ is illustrated by the help of 'Discriminant'. [$b^2-4ac>0$ $\Rightarrow$ Roots are real and distinct, $b^2-4ac=0\Rightarrow$ roots are real and equal, $b^2-4ac<0\Rightarrow$ on real roots].
- Examples about the use of 'Discriminant' to find out the nature of the roots of a quadratic equation are illustrated.
- Relation between roots and coefficients of an algebraic equation are established. It is shown that for a quadratic equation $ax^2+bx+c=0$ *where* a,b,c *reals and* $a\neq 0$ if $\alpha,\ \beta$ be the roots then $\alpha+\beta=\dfrac{-b}{a}=-\dfrac{coeff.\ of\ x}{coeff.\ of\ x^2}$ *and* $\alpha\beta=\dfrac{c}{a}=\dfrac{const.\ term}{coeff.\ of\ x^2}$.
- Some examples are used to find out the roots in different conditions by the help of relation between roots and coefficients of a quadratic equation.
- Construction of quadratic equation with the help of its roots is shown by an example i.e. if $\alpha,\ \beta$ be the roots of a quadratic equation then the equation may be written as $x^2-(\alpha+\beta)x+\alpha\beta=0$.

Some suggestions are listed below for improvement for the textual representation.

- An example of quadratic equation in two variables may be sited for clear understanding of quadratic equation in one variable and more than one variable.
- Transformation of higher order equation into quadratic equation may be discussed properly. It is mentioned in an exercise only.
- A details discussion is necessary about why it is necessary to write $a\neq 0$ for $ax^2+bx+c=0,\ a,b,c$ *reals*.
- A clear discussion is necessary to clear the concept that 'a quadratic equation has exactly two roots'. Roots may not be real. Two possibilities are there, (a) both roots are real, or (b) both roots are not real (complex actually).
- From quadratic equation the roots can be determined; conversely if one knows the roots of a quadratic equation then respective quadratic equation may be constructed. A clear discussion about this concept should be explored. The concept is found in the example only.

To analyze the mathematical exercise represented in this unit it is observed that

- Routine exercises are included.
- Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.
- Questions are represented according to the structure of the text and these are sequentially arranged.
- Modification in word problem is necessary. Actually number of word problem is less. It is suggested to increase the number of word problems.

Table 1: Examples and exercises presented in Unit I.

Method	Examples	Exercises		
		Routine	Evaluate, Analyze and Reason mathematically	Word Problem
Factorization	17	23	28	10
Completing Square + Rule of Sridhar Acharya	20	14	0	9
Use of Discriminant and Relation between Roots and Coefficients	17	27		0

In this book, content of unit V (p. 77-99) is Ratio and Proportion.

Representation of the text in this unit is as follows:

- Concept of ration, antecedent term, consequent term, ratio of equality, ratio of inequality, ratio of greater inequality, ratio of less inequality, inverse ratio, compound or mixed ratio are expressed by suitable example.
- Applications of those concepts are illustrated by examples.
- Examples of ratio with negative term and its real life example is illustrated.
- Concept of proportion is introduced.
- Concept of extreme proportional and mean proportional are explored by suitable example.
- Calculation of one of the 1st, 2nd, 3rd, 4th proportional with the help of other three proportional and the law of proportionality. Construction of equation and its solution is used to find unknown proportional.
- Terms like invertindo, componendo, dividendo,

componendo and dividendo are introduced and explained with the help of proper example.
- Applications of those concepts are illustrated with suitable examples.

For improvement of this unit, it is suggested that revision is necessary for real life problems. Real life examples are not used properly. Actually, I am recommending that examples from real life (word problems) should be increased.

To analyze the mathematical exercise represented in this unit it is observed that

- Routine exercises are included properly.
- Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.
- Questions are represented according to the structure of the text and these are sequentially arranged.
- Modification in word problem is necessary. Actually there is no real world problem. It is suggested to incorporate word problems.

Table 2: Examples and exercises presented in Unit V.

Subject matter	Examples	Exercises		
		Routine	Evaluate, Analyze and Reason mathematically	Word Problem
Ratio	15	12	14	0
Proportion	33	19	7	0
Identity related topics and their use	18	20	30	0

Other aspects which are mentioned in methodology may be discussed as

- Mathematical text used in the text book is clear to the reader. It is also found that there is no computational error in the algebra text.
- Mathematical concepts are presented sequentially. Appropriate teaching learning methods are applicable but clearness of some definitions is required.
- Development of content of algebra is already discussed. One may consider analysis and synthesis and discovery as teaching method. Problem solving method is also very much effective for solution of the problems given in this unit.
- Language of the book is very simple, clear. Explanations are also very simple.
- Routine exercises are appropriate for the concept presentation. But it is necessary to modify the exercises which will evaluate the students or will develop the power of analysis or strengthen mathematical reasoning.
- No historical examples or exercises are included in above mentioned units.

Table 3: Textual Analysis summary of unit I, Quadratic equation with one variable of tenth grade mathematics textbook of WBBSE.

Name of the Book : Ganit Prakash			Class: X
Number and Name of the Unit: I, Quadratic equation with one variable			Pages: 1-30
Analyzing Criteria	**Presentations and suggestions**		
Consistency and clearness of Mathematical content	Presentation	Textual presentation is consistent and clear to the reader. There is no computational or logical error.	
	Suggestion		
Mathematical theoretical aspects (Kernels)	Presentation	Good	
	Suggestion	A clear discussion is necessary to clear the concept that 'a quadratic equation has exactly two roots'. Roots may not be real. Two possibilities (a) both roots are real, or (b) both roots are not real (complex actually).	
Mathematical content development and connections	Presentation	Good	
	Suggestion	*An example of quadratic equation in two variables may be sited for clear understanding of quadratic equation. *From quadratic equation the roots can be determined; conversely if one knows the roots of a quadratic equation then respective quadratic equation may be constructed. A clear discussion about this concept should be explored. The concept is found in the examples.	
Mathematical representations and applications	Presentation		
	Suggestion	A details discussion is necessary about why it is necessary to write $a \neq 0$ for $ax^2 + bx + c = 0,\ a, b, c \quad reals$.	
Language used	Presentation	Language is simple and clear to the reader.	
	Suggestion		
Routine exercises	Presentation	Good	
	Suggestion		
Exercise related to Analysis, Synthesis Evaluation and Skill	Presentation	Good	
	Suggestion		
Real World related exercises (Word problems)	Presentation	Modification necessary	
	Suggestion	Addition of real world problems is necessary.	

Table 4: Textual Analysis summary of unit V: Ratio and Proportion of tenth grade mathematics textbook of WBBSE.

Name of the Book : Ganit Prakash			Class: X
Number and Name of the Unit: V: Ratio and Proportion			Pages: 77-99
Analyzing Criteria	**Presentations and suggestions**		
Consistency and clearness of Mathematical content	Presentation	Textual presentation is consistent and clear to the reader. There is no computational or logical error.	
	Suggestion		
Mathematical theoretical aspects (Kernels)	Presentation	Good	
	Suggestion		
Mathematical content development and connections	Presentation	Good	
	Suggestion		
Mathematical representations and applications	Presentation	Good	
	Suggestion	Real life examples are not used properly.	
Language used	Presentation	Language is simple and clear to the reader.	
	Suggestion		
Routine exercises	Presentation	Appropriate	
	Suggestion		
Exercise related to Analysis, Synthesis Evaluation and Skill	Presentation	Good	
	Suggestion		
Real World related exercises (Word problems)	Presentation	No real world problem.	
	Suggestion	Word problems should be incorporated	

Concluding remarks

It is mentioned in the text book that the book is written on the basis of NCF 2005 and learning will be activity based. It is actually psychological to logical approach to relate mathematics to real world. Teachers are encouraged to help in constructing students' knowledge. It seems that the algebra content of the text book demands slide modifications. Lastly it is suggested that there should be a concept summary listed at the end of the text and an exercise containing problems on every concept at the extreme end of the units discussed before for correlation and evaluation of concepts.

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